Chapter 16

THE TIME VALUE OF MONEY AND NET PRESENT VALUE

A bird in the hand is worth two in the bush

For economic progress to be possible, there must be a universally applicable time value of money, even in a risk-free environment. This fundamental concept gives rise to the techniques of capitalisation, discounting and net present value, described below.

These are more than just tools, but actual reflexes that must be studied and acquired.

Section 16.1
CAPITALISATION

Consider an example of a businessman who invests €100,000 in his business at the end of 1999 and then sells it 10 years later for €1,800,000. In the meantime, he receives no income from his business, nor does he invest any additional funds into it. Here is a simple problem: given an initial outlay of €100,000 that becomes €1,800,000 in 10 years, and without any outside funds being invested in the business, what is the return on the businessman's investment?

His pro t after 10 years was €1,700,000 (€1,800,000 – €100,000) on an initial outlay of €100,000. Hence, his return was (1,700,000/100,000) or 1700% over a period of 10 years.

Is this a good result or not?

Actually, the return is not quite as impressive as it rst looks. To nd the annual return, our rst thought might be to divide the total return (1700%) by number of years (10) and say that the average return is 170% per year.

While this may look like a reasonable approach, it is in fact far from accurate. The value 170% has nothing to do with an annual return, which compares the funds invested and the funds recovered after 1 year. In the case above, there is no income for 10 years. Usually, calculating interest assumes a ow of revenue each year, which can then be re-invested, and which in turn begins producing additional interest.

To calculate returns over a period greater than one year, we cannot simply compare the end return to the initial outlay and divide by the number of years. This is erroneous reasoning.

There is only one sensible way to calculate the return on the above investment. First, it is necessary to seek the rate of return on a hypothetical investment that would generate income at the end of each year. After 10 years, the rate of return on the initial investment will have to have transformed €100,000 into €1,800,000. Further, the income generated must not be paid out, but rather it has to be reinvested (in which case the income is said to be **capitalised**).

Capitalising income means foregoing receipt of it. It then becomes capital and itself begins to produce interest during the following periods.

Therefore, we are now trying to calculate the annual return on an investment that grows from €100,000 into €1,800,000 after 10 years, with all annual income to be reinvested each year.

An initial attempt to solve this problem can be made using a rate of return equal to 10%. If at the end of 1999, €100,000 is invested at that rate, it will produce 10% × €100,000, or €10,000 in interest in 2000.

This €10,000 will then be added to the initial capital outlay and begin, in turn, to produce interest. (Hence the term "to capitalise," which means to add to capital.) The capital thus becomes €110,000 and produces 10% × €110,000 in interest in 2001, i.e. €10,000 on the initial outlay plus €1000 on the interest from 2000 (10% × €10,000). As the interest is reinvested, the capital becomes €110,000 + €11,000, or €121,000, which will produce €12,100 in interest in 2002, and so on.

If we keep doing this until 2009, we obtain a final sum of €259,374, as shown in the table.

Year	Capital at the beginning of the period (€) (1)	Income (€) (2) = 10% × (1)	Capital at the end of the period (€) = (1)+(2)
2000	100,000	10,000	110,000
2001	110,000	11,000	121,000
2002	121,000	12,100	133,100
2003	133,100	13,310	146,410
2004	146,410	14,641	161,051
2005	161,051	16,105	177,156
2006	177,156	17,716	194,872
2007	194,872	19,487	214,359
2008	214,359	21,436	235,795
2009	235,795	23,579	259,374

Each year, interest is capitalised and itself produces interest. This is called **compound interest**. This is easy to express in a formula:

$$V_{2000} = V_{1999} + 10\% \times V_{1999} = V_{1999} \times (1 + 10\%)$$

Which can be generalised into the following:

$$V_n = V_{n-1} \times (1 + r)$$

where V is a sum and r the rate of return.

Hence, $V_{2000} = V_{1999} \times (1 + 10\%)$, but the same principle can also yield:

$$V_{2001} = V_{2000} \times (1 + 10\%)$$
$$V_{2002} = V_{2001} \times (1 + 10\%)\dots$$
$$V_{2003} = V_{2002} \times (1 + 10\%)$$

All these equations can be consolidated into the following:

$$V_{2009} = V_{1999} \times (1 + 10\%)^{10}$$

Or, more generally:

$$V_n = V_0 \times (1 + k)^n$$

Capitalisation formula

where V_0 **is the initial value of the investment,** r **is the rate of return and** n **is the duration of the investment in years.**

This is a simple equation that gets us from the initial capital to the terminal capital. Terminal capital is a function of the rate, r, and the duration, n.

Now it is possible to determine the annual return. In the example, the annual rate of return is not 170%, but 33.5%[1] (which is not bad, all the same!). There-fore, *33.5%* is the rate on an investment that transforms €100,000 into €1,800,000 in 10 years, with annual income assumed to be reinvested every year at the same rate.

1 $33.5\% = \left(\dfrac{1{,}800{,}000}{100{,}000}\right)^{\frac{1}{10}} - 1.$

To calculate the return on an investment that does not distribute income, it is possible to reason by analogy. This is done using an investment that, over the same duration, transforms the same initial capital into the same terminal capital and produces annual income reinvested at the same rate of return. At 33.5%, annual income of €33,500 for 10 years (plus the initial investment of €100,000 paid back after the 10th year) is exactly the same as not receiving any income for 10 years and then receiving €1,800,000 in the 10th year.

DISCOUNTING AND CAPITALISATION AT 15%
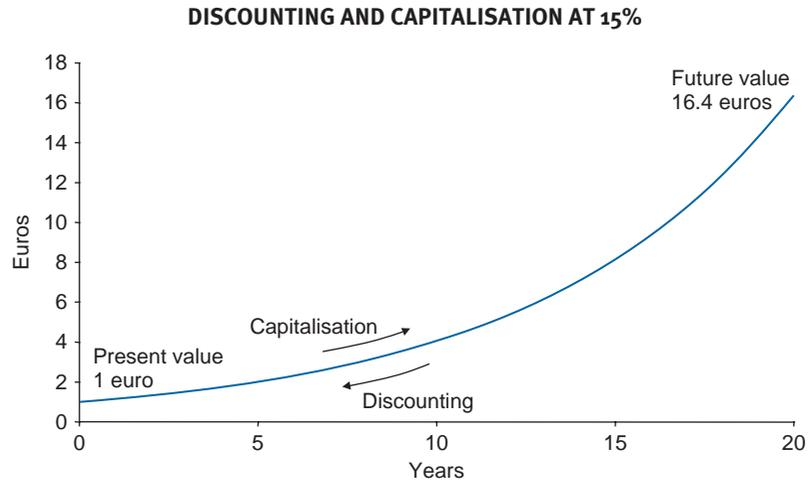
Over a long period of time, the impact of a change in the capitalisation rate on the terminal value looks as follows:

VALUE OF 1 EURO CAPITALISED AT VARIOUS RATES
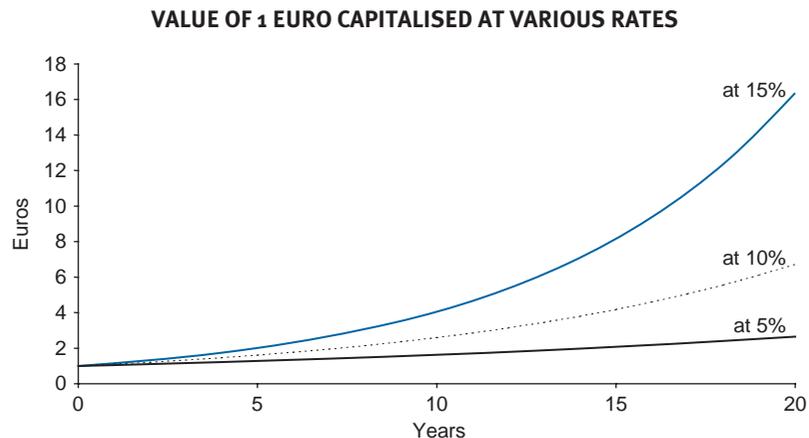
This increase in terminal value is especially important in equity valuations. The example we gave earlier of the businessman selling his company after 10 years is typical. The lower the income he has received on his investment, the more he would expect to receive when selling it. Only a high valuation would give him a return that makes economic sense.

The lack of intermediate income must be offset by a high terminal valuation. The same line of reasoning applies to an industrial investment that does not produce any income during the rst few years. The longer it takes it to produce its rst income, the greater that income must be in order to produce a satisfactory return.

Tripling one's capital in 16 years, doubling it in 10 years, or simply asking for a 7.177% annual return all amount to the same thing, since the rate of return is the same.

No distinction has been made in this chapter between income, reimbursement and actual cash flow. Regardless of whether income is paid out or reinvested, it has been shown that the slightest change in the timing of income modifies the rate of return.

To simplify, consider an investment of 100, which must be paid off at the end of year 1, with an interest accrued of 10. Suppose, however, that the borrower is negligent and the lender absent-minded, and the borrower repays the principal and the interest one year later than he should. The return on a well-managed investment that is equivalent to the so-called 10% on our absent-minded investor's loan can be expressed as:

$$V = V_0 \times (l + r)^2$$

$$\text{or} \quad 110 = 100 \times (1 + r)^2$$

$$\text{hence} \quad r = 4.88\%$$

This return is less than half of the initially expected return!

It is not accounting and legal appearances that matter, but rather actual cash flows.

Any precise financial calculation must account for cash flow exactly at the moment when it is received and not just when it is due.

Section 16.2
DISCOUNTING

1/ WHAT DOES IT MEAN TO DISCOUNT A SUM?

To discount means to calculate the present value of a future cash flow.

Discounting into today's euros helps us compare a sum that will not be produced until later. Technically speaking, what is discounting?

To discount is to "depreciate" the future. It is to be more rigorous with future cash flows than present cash flows, because future cash flows cannot be spent or invested immediately. First, take tomorrow's cash flow and then apply to it a multiplier coefficient below 1, which is called a discounting factor. The discounting factor is used to express a future value as a present value, thus reflecting the depreciation brought on by time.

Consider an offer whereby someone will give you €1000 in 5 years. As you will not receive this sum for another 5 years, you can apply a discounting factor to it, for example, 0.6. The present, or today's, value of this future sum is then 600. Having discounted the future value to a present value, we can then compare it to other values. For example, it is preferable to receive 650 today rather than 1000 in 5 years, as the present value of 1000 5 years out is 600, and that is below 650.

Discounting makes it possible to compare sums received or paid out at different dates.

Discounting is based on the time value of money. After all, "time is money". Any sum received later is worth less than the same sum received today.

Remember that investors discount because **they demand a certain rate of return**. If a security pays you 110 in one year and you wish to see a return of 10% on your investment, the most you would pay today for the security (i.e., its present value) is 100. At this price (100) and for the amount you know you will receive in 1 year (110), you will get a return of 10% on your investment of 100. However, if a return of 11% is required on the investment, then the price you are willing to pay changes. In this case, you would be willing to pay no more than 99.1 for the security because the gain would have been 10.9 (or 11% of 99.1), which will still give you a nal payment of 110.

Discounting is calculated with the required return of the investor. If the investment does not meet or exceed the investor's expectations, he will forego it and seek a better opportunity elsewhere.

Discounting converts a future value into a present value. This is the opposite result of capitalisation.

Discounting converts future values into present values, while capitalisation converts present values into future ones. Hence, to return to the example above, €1,800,000 in 10 years discounted at 33.5% is today worth €100,000. €100,000 today will be worth €1,800,000 when capitalised at 33.5%. over 10 years

Discounting and capitalisation are thus two ways of expressing the same phenomenon: the time value of money.
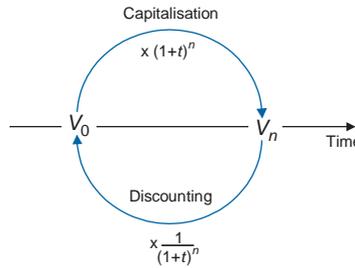
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2/ DISCOUNTING AND CAPITALISATION FACTORS

To discount a sum, the same mathematical formulas are used as those for capitalising a sum. Discounting calculates the sum in the opposite direction to capitalising.

To get from €100,000 today to €1,800,000 in 10 years, we multiply €100,000 by $(1 + 0.335)^{10}$, or 18. The number 18 is the **capitalisation factor**.

To get from €1,800,000 in 10 years to its present value today, we would have to multiply €1,800,000 by $1/(1 + 0.335)^{10}$, or 0.056.

0.056 is the **discounting factor**, which is the inverse of the coef cient of capitalisation. The present value of €1,800,000 in 10 years at a 33.5% rate is €100,000.

More generally:

Discounting formula

$$V_0 = \frac{V_n}{(1 + r)^n}$$

Which is the exact opposite of the capitalisation formula.

$1/(1 + r)^n$ is the **discounting factor**, which depreciates V_n and converts it into a present value V_0. It remains below 1 as discounting rates are always positive.

Section 16.3

PRESENT VALUE AND NET PRESENT VALUE OF A FINANCIAL SECURITY

In the introductory chapter of this book, it was explained that a financial security is no more than a stream of future cash flow, to which we can then apply the notion of discounting. So, without being aware of it, you already knew how to calculate the value of a security!

1/ FROM THE PRESENT VALUE OF A SECURITY . . .

The Present Value (PV) of a security is the sum of its discounted cash flows ; i.e.:

$$PV = \sum_{n=1}^{N} \frac{F_n}{(1+r)^n}$$

where F_n are the cash flows generated by the security, r is the applied discounting rate and n is the number of years for which the security is discounted.

All securities also have a **market value**, particularly on the secondary market. Market value is the price at which a security can be bought or sold.

Net present value (NPV) is the difference between present value and market value (V_0):

$$NPV = \sum_{n=1}^{N} \frac{F_n}{(1+r)^n} - V_o$$

If the net present value of a security is greater than its market value, then it will be worth more in the future than the market has presently valued it. Therefore you will probably want to invest in it, i.e. to invest in the upside potential of its value.

If, however, the security's present value is below its market value, you should sell it at once, for **its market value is sure to diminish.**

2/ . . . TO ITS FAIR VALUE

If an imbalance occurs between a security's market value and its present value, efficient markets will seek to re-establish balance and reduce net present value to zero. Investors acting on efficient markets seek out investments offering positive net present value, in order to realise that value. When they do so, they push net present value towards zero, ultimately arriving at the fair value of the security.

In efficient, fairly valued markets, net present values are zero, i.e. market value is equal to present value.

3/ APPLYING THE CONCEPT OF NET PRESENT VALUE TO OTHER INVESTMENTS

Up to this point, the discussion has been limited to nancial securities. However, the concepts of present value and net present value can easily be applied to any investment, such as the construction of a new factory, the launch of a new product, the takeover of a competing company or any other asset that will generate positive and/or negative cash ows.

The **concept of net present value** can be interpreted **in three different ways**:

1 **the value created by an investment** – for example, if the investment requires an outlay of €100 and the present value of its future cash ow is €110, then the investor has become €10 wealthier.
2 **the maximum additional amount that the investor is willing to pay to make the investment** – if the investor pays up to €10 more, he/she has not necessarily made a bad deal, as he/she is paying up to €110 for an asset that is worth 110.
3 **the difference between the present value of the investment (€110) and its market value (€100).**

Section 16.4
THE NPV DECISION RULE

Calculating the NPV of a project is conceptually easy. There are basically two steps to be followed:

1 write down the net cash ows that the investment will generate over its life;
2 discount these cash ows at an interest rate that re ects the degree of risk inherent in the project.

The resulting sum of discounted cash ows equals the project's net present value. The **NPV decision rule** says to invest in projects when the present value is positive (greater than zero):

$$NPV > 0 \quad \text{invest}$$

$$NPV < 0 \quad \text{do not invest}$$

The NPV rule implies that rms should invest when the present value of future cash in ows exceeds the initial cost of the project. Why does the NPV rule lead to good investment decisions? The rm's primary goal is to maximise shareholder wealth. The discount rate r represents the highest rate of return (opportunity cost) that investors could obtain in the marketplace in an investment with equal risk. When the NPV of cash ow equals zero, the rate of return provided by the investment is exactly equal to investors' required return. Therefore, when a rm nds a project with a positive NPV, that project will offer a return exceeding investors' expectations.

Although this section will highlight many of the advantageous qualities of the NPV approach, there are also a few weaknesses that bear mentioning:

- it is less intuitive than other methodologies, such as the payback rule or the accounting return rule, which are presented in Chapter 18;
- it does not take into account the value of managerial exibility, in other words the options that managers can exploit after an investment has been made in order to increase its value; and
- the NPV has a major competitor in the Internal Rate of Return (IRR), whose use seems more widespread among corporations. In most cases, the two decision rules give the same information, but the IRR is more appealing to managers because it delivers a number that is more easily interpreted.

Section 16.5
WHAT DOES NET PRESENT VALUE DEPEND ON?

While net present value is obviously based on the amount and timing of cash ows, it is worth examining how it varies with the discounting rate.

The higher the discounting rate, the more future cash ow is depreciated and, there-fore, the lower is the present value. **Net present value declines in inverse proportion to the discounting rate**, thus re ecting investor demand for a greater return (i.e. greater value attributed to time).

Take the following example of an asset (e.g. a nancial security or a capital investment) whose market value is 2 and whose cash ows are as follows:

Year	1	2	3	4	5
Cash flow	0.8	0.8	0.8	0.8	0.8

A 20% discounting rate would produce the following discounting factors:

Year	1	2	3	4	5
Discounting factor	0.833	0.694	0.579	0.482	0.402
Present value of cash flow	0.67	0.56	0.46	0.39	0.32

As a result, the present value of this investment is about 2.4. As its market value is 2, its net present value is approximately 0.4.

If the discounting rate changes, the following values are obtained:

Discounting rate	0%	10%	20%	25%	30%	35%
Present value of the investment	4	3.03	2.39	2.15	1.95	1.78
Market value	2	2	2	2	2	2
Net present value	2	1.03	0.39	0.15	−0.05	−0.22

Which would then look like this graphically

NET PRESENT VALUE AND THE DISCOUNT RATE
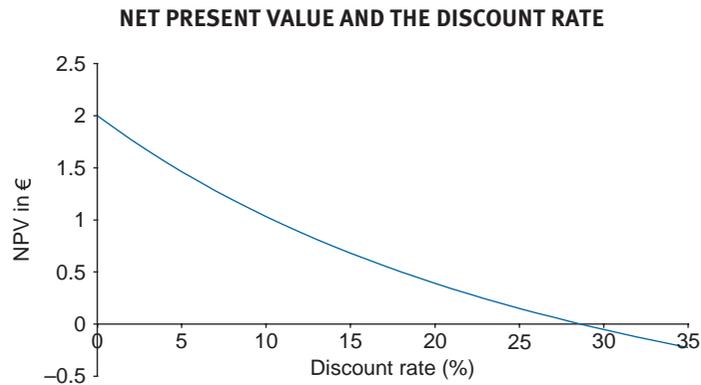
The present value and net present value of an asset vary in inverse proportion to the discounting rate.

Section 16.6
SOME EXAMPLES OF SIMPLIFICATION OF PRESENT VALUE CALCULATIONS

For those occasions when you are without your favourite spreadsheet program, you may find the following formulas handy in calculating present value.

1/ THE VALUE OF AN ANNUITY F OVER N YEARS, BEGINNING IN YEAR 1

$$PV = \frac{F}{(1+r)} + \frac{F}{(1+r)^2} + \ldots + \frac{F}{(1+r)^N}$$

or:

$$PV = F \times \left(\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^N} \right)$$

For the two formulas above, the sum of the geometric series can be expressed more simply as:

$$PV = \frac{F}{r} \times 1 \quad \frac{1}{(1+r)^N}$$

or:

$$PV = F \times \frac{1}{r} \quad \frac{1}{r \times (1+r)^N}$$

So, if $F = 0.8$, $k = 20\%$ and $N = 5$, then the present value is indeed 2.4.

Further, $(1/r) \times (1-(1+\rho)^{-N})$ is equal to the sum of the rst n discounting factors.

The term used to compute the value of the stream of constant payments, F, for N years is called an **annuity factor**. An example of an annuity is the coupon part of a bond with equal annual payments. The annuity factor in the current example is 2.9906. The table at the end of this book gives the value of these factors for a range of interest rates, r, and maturity dates, N.

For simplicity, we refer to the annuity factor as:

$$A_r^N$$

2/ THE VALUE OF A PERPETUITY

A **perpetuity** is a constant stream of cash ows without end. By adding this feature to the previous case, the formula then looks like this:

$$PV = \frac{F}{(1+r)} + \frac{F}{(1+r)^2} + \ldots + \frac{F}{(1+r)^n} + \ldots + \ldots$$

As n approaches in nity, this can be shortened to the following:

$$PV = \frac{F}{r}$$

The present value of a €100 perpetuity discounted back at 10% per year is thus:

$$PV = 100/0.10 = €1000$$

A €100 perpetuity discounted at 10% is worth €1000 in today's euros. If the investor demands a 20% return, the same perpetuity is worth €500.

3/ THE VALUE OF AN ANNUITY THAT GROWS AT RATE g FOR n YEARS

In this case, the F_0 cash ow rises annually by g for n years.

Thus:

$$VA = \frac{F_0 \times (1+g)}{(1+r)} + \ldots + \frac{F_0 \times (1+g)^n}{(1+r)^n}$$

or:

$$PV = \frac{F_0 \times (1+g)}{(r-g)} \times 1 \frac{(1+g)^n}{(1+r)^n}$$

Note: the rst cash ow actually paid out is $F_0 \times (1+g)$

Thus, a security that has just paid out 0.8, and with this 0.8 growing by 10% each year for the four following years – has – at a discounting rate of 20%, a present value of:

$$PV = (0.8 \times (1+10\%)/(20\% - 10\%)) \times (1 - (1.10/1.20)^4) = 2.59$$

4/ THE VALUE OF A PERPETUITY THAT GROWS AT RATE g (GROWING PERPETUITY)

As n approaches in nity, the previous formula can be expressed as follows:

$$PV = \frac{F_0 \times (1+g)}{r-g} = \frac{F_1}{r-g}$$

As long as $r > g$.

The present value is thus equal to the next year's cash ow divided by the difference between the discounting rate and the annual growth rate.

For example, a security with an annual return of 0.8, growing by 10% annually to in nity has, at a rate of 20%, a PV $= 0.8/(0.2 - 0.1) = 8.0$.

5/ WHEN CASH FLOWS RISE AT DIFFERENT RATES

This formula is useful when the growth rate is very high at the beginning of the period of projection, i.e. higher than the discounting rate, and then gradually declines. (After all, not even trees can grow forever!)

Over three periods lasting n_1 years, n_2 years, and up to in nity, cash ow rises by g_1 for $n_1 - 1$ years, then by g_2 for n_2 years and then by g_3 to in nity. Present value is then equal to:

$$PV = F_1 \times \left[\frac{1 - \left(\frac{1+g_1}{1+k}\right)^{n_1}}{k - g_1} + \frac{(1+g_1)^{n_1-1}}{(1+k)^{n_1}} \times (1+g_2) \times \frac{1 - \left(\frac{1+g_2}{1+k}\right)^{n_2}}{k - g_2} \right.$$

$$\left. + \frac{\frac{(1+g_1)^{n_1} \times (1+g_2)^{n_2} \times (1+g_3)}{k - g_3}}{(1+k)^{n_1+n_2}} \right]$$

For example, if the rst year's cash ow is 10.75, and it grows by 15% annually for the next ve years, then by 9% annually for the ve following years, and nally by 2.5% from year 11 onwards, the value of this asset is then about 270, based on a 10% discounting rate.

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Section 16.7
SPECIAL **NPV** TOPICS

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The following are additional topics for consideration when calculating NPV.

1/ CAPITAL RATIONING AND THE PRESENT VALUE INDEX

Sometimes there is a strict capital constraint imposed on the rm, and it is faced with more NPV positive projects than it can afford. In order to determine which project to pursue, the best formula to use is the **Present Value Index (PVI)**. This is the present value of cash in ows divided by the present value of cash out ow:

$$PVI = \frac{\text{Present value of in ows}}{\text{Present value of out ows}}$$

By using the Present Value Index, nancial managers can rank the different projects and then select the investment with the highest PVI – that is, the project with the highest NPV relative to the present value of out ows. After making this selection, if the total amount of capital available has not been fully exhausted, the managers should then invest in the project with the second-highest PVI, and so on until no more capital remains to invest.[2]

More generally, the objective is to compare all combinations of x projects that meet the budget and nd the one that maximises the *weighted average PVI:*

$$PVI = \frac{PV \text{ out ows Project } A}{\text{Total funds available}} \times (PVI_A) + \ldots + \frac{PV \text{ out ows Project } X}{\text{Total funds available}} \times (PVI_X)$$

2/ PROJECTS WITH DIFFERENT LIVES

At other times, it is necessary to compare two (or more) similar projects that have positive net present value and similar PVI but with different operating costs and lasting for dif- ferent time periods. The NPV rule would suggest undertaking the project with the lower cost. However, this conclusion could lead to the wrong decision if the lower cost project needs to be replaced before an alternative investment.

When we have to choose between two projects of two different lengths, we must reason on an equal life basis, which can be done using the **Equivalent Annual Cost (EAC) method.** This technique is based on the following steps:

1   calculate the present value of the costs of each of the two projects;
2   determine the equivalent annual cost of the single payment represented by the PV of the costs of the two projects. We can use the tables of the present value of an annuity; and
3   choose the project with the lower equivalent annual cost.

Consider the following example. A company can undertake two (mutually exclusive) projects with different initial outlays and periodical costs. The rst project (*A*) lasts for 3 years and the second project (*B*) lasts for 4 years. The discount rate is 10%.

	Initial investment	Annual costs				PV of costs
	0	1	2	3	4	
Project *A*	400	100	100	100		€ 590
Project *B*	500	80	80	80	80	€ 685

The PV of the total costs is higher for the project *B*. A naive approach would be to select *A* because of the lower PV of the costs. However, project *A* has a shorter life, so perhaps it will need to be replaced earlier than project *B*.

Therefore, a different approach is required. By using annuities, and equating the PV of the total costs at date 0 with maturities of 3 years and 4 years, the following results are obtained:

PV of costs	Anuity factor	Annual equivalent cost
€ 590	2.4869	€ 237
€ 685	3.1699	€ 216

The annual equivalent cost for project B is €216, whereas for project A it is €237. Therefore, the manager should select project B.

The EAC converts total present value of costs to a cost per year. The reader should immediately realise that this is equivalent to a fair *rental payment* – i.e. a payment that the manager should afford periodically if he decides to rent the machine rather than buy it.

Comparing annual equivalent costs should take into account two nal caveats:

1   The use of EAC for comparison of costs should always be done in *real* terms. The nominal procedure could in fact give incorrect rankings with high in ation rates.

2   The use of EAC is useless if there is no replacement expected at different future dates.

SUMMARY

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Capitalisation involves foregoing immediate spending of a given sum of money. By using the interest rate at which the money will be invested, the future amounts can be calculated. Thus, the future value of a sum of money can be determined by way of capitalisation.

Discounting involves calculating today's value of a future cash flow, what is known as the *present value*, on the basis of rates of return required by investors. By calculating the present value of a future sum, discounting can be used for comparing future cash flows that will not be received on the same date.

Discounting and capitalisation are two ways of expressing the same phenomenon: the time value of money.

Capitalisation is based on compound interest. $V_n = V_0 \times (1 + r)^n$ where V_0 is the initial value of the investment, k is the rate of return, n is the duration of the investment in years, $(1 + r)^n$ is the capitalisation factor, and V_n the terminal value.

Discounting is the inverse of capitalisation. It is important to note that any precise financial calculation must account for cash flows at the moment when they are received or paid, and not when they are due.

Net present value (NPV) is the difference between present value and the value at which the security or share can be bought. Net present value measures the creation or destruction of value that could result from the purchase of a security or making an investment. When markets are in equilibrium, net present values are usually nil.

Changes in present value and net present value move in the opposite direction from changes in discount rates. The higher the discount rate, the lower the present value and net present value, and vice versa.

In many cases, calculating present value and net present value can be made a lot simpler through *ad hoc* formulas.

Finally, there are two special situations where the NPV rule is still valid: when projects have different lives and when there is capital rationing. If two (or more) projects have different lives, the manager should use the Equivalent Annual Cost; if there is capital rationing, then it could be helpful to use the present value index.

1/What is discounting?

2/Why should we discount?

3/What is the discount factor equal to?

4/On what should you base a choice between two equal discounted values?

5/What is the simple link between the discount factor and the capitalisation factor?

6/Why are capitalisation factors always greater than 1?

7/Why are discount factors always less than 1?

8/Should you discount even if there is no inflation and no risk? Why?

9/Why does the graph on capitalisation show curves and not lines?

10/Belgacom pays out big dividends. Should its share price rise faster or slower than the share price of Ryanair which doesn't pay out any dividends? Why? Would it be better to have Belgacom stock options or Ryanair stock options? Why?

11/What is net present value equal to?

12/The higher the rates of return, the larger present values will be. True or false?

13/What mechanism pushes market value towards present value?

14/Can net present value be negative? What does this mean?

15/What does the discount rate correspond to in formulas for calculating present value and net present value?

16/Are initial flows on an investment more often positive or negative? What about for final cash flows?

17/A market is in equilibrium when present values are nil and net present values are positive. True or false?

18/For the investment in Section 16.2, what is the maximum discount rate above which it would not be worthwhile for the investor?

19/Can the growth rate to infinity of a cash flow be higher than the discount rate? Why?

20/ Could an investment made at a negative net present value result in the creation of value?

21/ Would you be more likely to find investments with positive present value on financial markets or on industrial markets? Why?

EXERCISES

1/ What is the present value of €100 received in 3 years at 5%, 10% and 20%?

2/ What is the present value at 10% of €100 received in 3 years, 5 years and 10 years? What are the discount factors?

3/ How much would €1000 be worth in 5 years, invested at 5%, 10% and 20%. Why is the sum invested at 20% not double that invested at 10%?

4/ How much would €1000 be worth in 5 years, 10 years and 20 years if invested at 8%? Why is the sum invested for 20 years not double that invested for 10 years?

5/ You are keen to obtain a helicopter pilot's licence. A club offers you lessons over two years, with a choice between the following payment terms:

- ○ you can either pay the full fees (€10,000) immediately with a 10% discount; or
- ○ you can make two equal annual payments, the first one due immediately.

At what interest rate would these two options work out at the same cost?

6/ What interest rate would turn 110 into 121 in 1 year?

7/ How much would you have to invest today to have 100 in 8 years if the interest rate was 5%? What is the capitalisation factor?

8/ At 7%, would you rather have €100 today or €131.1 in 4 years' time? Why?

9/ Show that in order to double your money in one year, the interest rate would have to be around 75%/year.

10/ Show that in order to treble your money in N years, the interest rate would have to be around 125%/N.

11/ You are only prepared to forgo immediate spending if you get a 9% return on your investment. What would be the top price you would be prepared to pay for a security today that would pay you 121 in 2 years? If other investors were asking for 8%, what would happen?

12/ If instead of throwing his 30 pieces of silver away in 33 AD, Judas had invested them at 3% per annum, how much would his descendants get in 2009? Explain your views.

13/ You have the choice between buying a Francis Bacon painting for €100,000 which will be worth €125,000 in 4 years, and investing in government bonds at 6%. What would your choice be? Why?

14/ Given the level of risk, you require an 18% return on shares in Amazon.com. No dividends will be paid out for 5 years. What is the lowest price you could sell them at in 4 years' time, if you bought them for $14 a share today.

15/ Ten years from now, at what price should you sell shares in Belgacom, which pays a constant annual dividend to infinity of €2.4, in order to get a 7% return? The share price today is €25.9.

16/ Assume that a share in Zaleski has a market value of 897, with the following cash flow schedule:

Year	1	2	3	4	5
Cash flow	300	300	300	300	300

Calculate the NPV of the share at 5%, 10%, 20% and 25%. Plot your answers on a graph.

17/ What is the present value at 10% of a perpetual income of 100? And a perpetual income of 100 rising by 3% every year from the following year?

18/ What is the present value at 10% of €100 paid annually for 3 years? Same question for a perpetual income.

19/ An investment promises 4 annual payments of €52 over the next 4 years. You require an 8% return. How much would you be prepared to pay for this asset? The share is currently trading at €165. Would you be prepared to buy or to sell? Why? If you buy at that price, how much will you have gained? Will the rate of return on your investment be greater or less than 8%? Why? If you buy at €172, what will your return on this investment be? Why?

20/ Show that at 8% there is little difference between the value of a perpetual income and that of a security that offers a constant annual income equal to that of the perpetual income for only 40 years. Show that this will not be true if the rate of return is 15%.

21/ You have the opportunity to buy the right to park in a given parking place for 75 years, at a price of €300,000. You could also rent a parking place for €2000 a year, revised upwards by 2% every year. If the opportunity cost is 5%, which would you choose?

22/ You are the proud owner of the TV screening rights of the film *Singing in the Rain*. You sell the rights to screen the film on TV once every 2 years, for €0.8m. What is the value of your asset? The film has just been screened. You make the assumption that screenings will be possible for 30 years or in perpetuity. The discount rate is 6%.

23/ You have found your dream house and you have the choice between renting it with a lease in perpetuity for €12,000 or buying it. At what purchase price would you be better off renting, if the loan you needed to buy the house costs you 7%, and the rent increases by 3% per year?

24/ Your current after-tax annual income is €50,000, which should increase by 4% per year until you retire. You believe that if you interrupt your professional career for 2 years to do an MBA, you could earn €65,000 after tax per year, with an annual increase of 5% until you retire. What is your present value if you retire in 40 years' time, and the discount rate is 4%? If the total cost of the MBA is €50,000 payable immediately, what is the net present value of this investment? Is it worth doing an MBA?

Questions

1/ *Converting a future value into a present value.*
2/ *So as to be able to compare a future value and a present value of a future inflow.*
3/ $1/(1 + t)^n$.
4/ *If the present values are equal, it makes no difference.*
5/ *One is the opposite of the other.*
6/ *Because interest rates are positive.*
7/ *Because interest rates are positive.*
8/ *Yes, because discounting is used to factor in an interest rate which remunerates the foregoing of immediate spending. Discounting is thus unrelated to inflation or risk.*
9/ *Because of capitalisation, which every year adds interest earned over the past year to the principal, and interest is earned on this interest in the future. This is called compound interest.*
10/ *The Ryanair's share price will have to rise more than that of Belgacom in order to make up for the lack of dividends. As stock options are options to buy shares at a fixed exercise price, their value will increase if the share price rises. So it would be better to have Ryanair's stock options.*
11/ *To the difference between the present value and the market value of an asset.*
12/ *False, the opposite is true.*
13/ *Arbitrage.*
14/ *Yes. The asset has been overvalued.*
15/ *To the required return on this asset.*
16/ *Negative, positive.*
17/ *False, the opposite is true. NPV = 0 and PV > 0.*
18/ *Around 28%.*
19/ *No, because growth is not a process that can continue endlessly!*
20/ *No, unless you've made an error in your calculations of the cash flows or underestimated them.*
21/ *In industrial markets because arbitrage operations take longer to execute than in financial markets (building a factory takes longer than buying a share) and, therefore, disequilibrium is more frequent.*

Exercises

1/ $100/1.05^3 = €86.4$; $100/1.1^3 = €75.1$; $100/1.2^3 = €57.9$.
2/ *€75.1; €62.1; €38.6; 0.751; 0.621; 0.386.*
3/ *€1276, €1611, €2488. Because the principal (€1000) remains the same and interest more than doubles as a result of the process of compound interest.*
4/ *€1469, €2159 and €4661. Because the principal (€1000) remains the same and interest more than doubles as a result of the process of compound interest.*
5/ *11.55% per year.*
6/ *10% (121/110 − 1).*
7/ *€67.7; €1.48.*
8/ *It makes absolutely no difference, because €100 capitalised at 7% a year would be worth €131.1 in 4 years.*
9/ *This is a good estimate. Over 5 years, a sum doubles at 14.87%, and 75%/5 = 15%.*
10/ *This is a good estimate. Over 5 years, a sum trebles at 24.57%, and 125%/5 = 25%.*
11/ *At 101.8 other investors are prepared to pay 103.7 and you cannot buy this security.*
12/ 6.97×10^{26} *pieces of silver (0.697 billion billion billion pieces of silver!). Although mathematically possible, Judas' descendants would be unlikely to get anything at all,*

given the wars, revolutions, periods of inflation, state bankruptcies, etc. that have occurred since 33 AD!

13/ €100,000 at 6% will be worth €126,248 in 4 years, which is more than €125,000, but if you're an art lover, it might be worth foregoing €1248 for the pleasure of admiring a Francis Bacon in the comfort of your own home for 4 years. There's more to life than money!

14/ $14 \times 1.18^4 = €27$.

15/ €137, because the whole of the return on this share is in the dividend. $7\% \times €137 = €9.6$.

16/ 402; 240; 109; 0; −90.

17/ 1000; 1429.

18/ €248.7; €1000.

19/ €172. Buy, because its present value is higher than its market value. €7. Greater than 8%, because at 8% it is worth €172, so if I buy at €165, I'll earn more. 8%.

20/ With income of 100, you get: 1250 and 1192, a difference of 5%. At 15%: 666.7 and 664.2, a difference of 0.4%. Barring other factors, income over a period exceeding 40 years no longer has a significant impact on present value.

21/ €2000 over 75 years growing at 2% would be worth €59,086, so it would be better to buy.

22/ €5.34m, €6.47m.

23/ €288 767.

24/ €1,923,077, €662,470, yes.

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The pioneering works on the net present value rule are:

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F. Lutz, V. Lutz, *The Theory of the Investment of the Firm*, Princeton University Press, 1951.

J. Tobin, Liquidity preference as behaviour towards risk, *Review of Economic Studies*, February 1958.

There are a number of financial calculation workbooks available which will help you get to grips with discounting calculations. You can also read:

T. Copeland, F. Weston, *Financial Theory and Corporate Finance*, Addison Wesley, 1987.

You could also consult:

E. Fama and M. Miller, *The Theory of Finance*, Rinehart and Winston, 1972.